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# A user's guide: The Adams-Novikov $E_2$-term for Behrens' spectrum $Q(2)$ at the prime 3

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## 4. Colloquial summary

In this summary I will—in the context of my paper, as much as possible—talk about what drew me to algebraic topology and homotopy theory, and why I find them exciting! My somewhat circuitous path through graduate school is a big reason why I became a homotopy theorist, so this will be partly autobiographical.

**4.1. West coast.** Before moving to western New York to write the dissertation that gave rise to the paper addressed by this user's guide, I spent a few formative years in Los Angeles as a math graduate student at UCLA. Those years were formative for three reasons. First and most importantly, I met my wife in southern California. Second, I learned (often the hard way) how hard it is to pursue mathematics seriously. Third, and most relevant to this discussion, I was exposed to the beautiful ideas of number theory.

UCLA had a thriving number theory group while I was there, as it does now. Soon after I arrived I gravitated toward analytic number theory, which in many ways is far removed from algebraic topology. For example, algebraic topologists eat categories and functors for breakfast every morning, while analytic number theorists seem allergic. Of course, I knew very little math at that point so I was blissfully unaware of such distinctions between mathematical diets. The number theorist at UCLA that I gravitated toward the most taught topics in analytic number theory (e.g., modular and automorphic forms, $L$-functions) from a historical perspective, because he so admired the pioneers of the subject (Hecke, Maaß, Selberg, Ramanujan, etc.). I learned to admire them too, and I had a wonderful time studying mathematics though this "old school" lens. As time went on, however, certain life events made it tough to get a thesis project off the ground at UCLA. I eventually left the program without my PhD. But number theory had left its mark on me, and I had a faint hope that I was not completely done with it.

**4.2. East coast.** I became interested in algebraic topology shortly after leaving Los Angeles. This was partly the influence of some papers by Dan Freed I stumbled on that blended homotopy theory, geometry, and physics—a blend I found intriguing (and still do). It was also partly the influence of a lecture by Mike Hopkins at the 25th anniversary of MSRI that I happened to watch online. But mostly, I was just in the market for a fresh mathematical start. While I still appreciated number theory and analysis, I yearned for something completely new. Algebraic topology fit that bill. It was a choice made in order to survive as a fledgling mathematician, just as much as it was a choice made out of mathematical taste.

In 2009, Doug Ravenel allowed me to come to Rochester for the fresh start I needed. It was incredibly good fortune, especially because Ravenel held a learning seminar that first fall semester on topological automorphic forms. I remember sitting in that seminar on the first day and thinking, "wow." I could not believe that automorphic forms, these analytic objects that number theorists coveted and that I had ostensibly left behind at UCLA, suddenly appeared in my new mathematical world of algebraic topology. The seminar was a bridge connecting my past mathematical life to my current one. Everything was both familiar and fresh at the same time. Finally, things seemed to fall into place.

Ravenel's $TAF$ seminar led me to Behrens-Lawson manuscript on $TAF$, which in turn led me to Behrens' papers. That is how I became interested in the $Q$-spectra specifically.

**4.3. The number 504.** My excitement for algebraic topology generally, and for homotopy theory in particular, owes a lot to the number 504. If my interest in homotopy theory were an episode of Sesame Street, it would definitely be brought to you by the number 504. This is because 504 happens to be the size of the 11th stable stem: $\pi_{11}S \cong \mathbb{Z}/504$. In fact, this appearance of 504 is a consequence of Adams' work on the image of $J$, and it's also number-theoretic, because there is a precise sense in which the 504 we see in the stable stems is the same 504 that we see in the recipe for the Eisenstein series Fourier expansion

$$E_6 = 1 - 504\sum_{n=1}^{\infty} \sigma_5(n)q^{2n}.$$

Even so, it's not like the sizes of stems 0 through 10 give you any warning that 504 is coming up next. They, in fact, give no warning at all. And if we think about this in a more concrete geometric context, we can reasonably ask why it should be that there are exactly 504 ways to throw the 100-dimensional sphere inside the 89-dimensional sphere up to deformation! Why not 503, or 505, or a more sane quantity like 2 or 3? This particular mystery is what has always drawn me (and no doubt others) to homotopy theory, and I feel lucky to be able to think about such mysteries as part of my job.

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