Enchiridion: Mathematical User's Guides
mathusersguides.com
Volume 2 (September 2016)

A user's guide: Dynamics and fluctuations of cellular cycles on CW complexes

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4. Colloquial summary

Imagine standing on a busy street with cars driving by. If you own a business on the street, work road construction, are a city planner, or work in the government traffic office, then the rate and actual number of cars driving by could be of great interest to you. The rate at which cars drive by depends on a variety of different factors, all of which can be described in the language developed so far in this user guide. In this section, we explore this example in detail in an attempt to connect the ideas of [**CCK**] with a real-world example.

The rate at which cars pass by is determined by a number of different features. Some of these features are completely determined ahead of time, that is they can be planned for, whereas others cannot. For example, the speed limit, the time of day, day of the week, and which businesses are nearby will all affect the rate at which cars pass. If we are interested in the rate as precise function of time, meaning on the level of seconds, then the traffic signals will have a tremendous impact. There are also random effects for which we cannot prepare and are beyond our control. In this situation, random could mean someone oversleeping their alarm before work and having to speed, a broken traffic signal, or even more subtle, the different speeds at which different people drive. Furthermore, the actual road itself will factor into this rate as well. The more intersections a road has, the more traffic signals and possibly more pedestrian traffic there can be, decreasing the rate at which cars pass. On the other hand, a road with more lanes can increase this rate.

In terms of the language developed in the previous sections of this user guide, we have described deterministic and stochastic (or random) effects, both of which affect the rate at which cars pass by. The deterministic component consists of the steady pattern of the traffic light, which forces the traffic to move in a specific direction at certain times, and more generally, a person's desire to travel to a destination. The stochastic component is mostly due to human drivers. Different people drive differently. Some go fast as soon as they see the green light, whereas others drive slower and with more caution. The varying driving styles, together with possible pedestrian traffic, force the flow of traffic to be slightly randomized, even though the traffic signals generally govern the flow of cars. Also the topology of the road, such as the number of lanes and number of intersections, has a great impact on the rate.

The main result of [**CCK**] is a statement about currents or rates under two important limits. The adiabatic limit is one in which the parameters vary slower and slower in time. In the example of traffic, this amounts to keeping the traffic lights greener for longer and longer. This would make the flow of traffic much more regular and consistent, since the cars no longer have to stop. They just keep driving at their normal pace. The second limit is the low-noise limit, which takes the form of forcing the drivers to become more alike in their driving styles. Drivers will start to accelerate at the same pace, drive at the same speed, and drive less erratically. In a more modern situation, this could be achieved by placing more and more driverless cars on the road. This again serves to regulate the flow of cars. In these two situations, counting cars as they pass becomes much easier. Furthermore, it is much more believable that something meaningful about passing car rates can be said under these circumstances. The main quantization theorem of [**CCK**] metaphorically implies the number and rate of cars driving by will be the same for equal durations of time.

Higher dimensional currents can also be thought of in a similar example. In order to understand currents of extended objects, consider a team of runners moving along a marathon course. We are again measuring current by counting how many times the team runs by a particular spot. Each runner's desire to win serves as the deterministic component, driving them all. The way in which they run, at different paces, and in different ways around the course causes random behavior. The layout of the course in terms of turns and possible junctions plays the role of topology. The course can have forks and places where the path splits, but ultimately all paths lead to the finish line. Suppose the team comes to a fork, and half run down one path and half down the other path. Each path was crossed by half the team, even though the team as a whole passed through the fork. Therefore, the 'current generated' by the team would be half on each leg of the fork, resulting in rational current. Note that any individual runner will only cross a single path of the fork, and therefore cross each leg either zero or one times, yielding integer current. While this is not precisely how rational quantization occurs in higher dimensions, it is the additional complexity of an extended object which gives rise to rational current for both our example and empirical currents.

References

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